Precalculus
A Modeling Approach

Claudinna Rowley, Math Professor

Issues with Precalculus

- Precalculus has a high DFW rate
- Precalculus is considered a “barrier course”
- There is incredible number of topics
- Topics are not cohesive and difficult to manage

My Strategy: Create a Quality Course

- Reduce the Number of Topics
  - Currently: 9 chapters - 47 sections
  - Calculus I: 5 chapters (1 review chapt) - 25 sections
- A Modeling Approach
  - 5 Models: Polynomial, Rational, Exponential, Trigonometric, Other
  - Start with real-world data
  - Create a model equation using technology
  - Use the model equation to analyze precalculus concepts
  - Write results in complete sentences using correct units
  - Represent the data and model graphically using correct scale and labels

Why a Modeling Approach?

- Starts with the concrete and moves to the abstract
- Makes the material interesting, applicable, relevant and cohesive
- Allows students to become problem solvers

Exponential Model

Below is a chart which shows the number of COVID-19 cases world-wide every two months from February to October 2020. The data is based on data from the World Health Organization (WHO) website.

Month	Feb Month2	Apr Month4	Jun Month6	Aug Month8	Oct Month10
Number of Cases	1300	75,000	123,000	297,000	301,000

a) Use Desmos (Log Mode on) to determine the exponential model $y = ab^x$ which represents all of the data. Enter 2 for Feb, 4 for April, etc. Round a to a whole number and b to 4 decimal places. Use N and t as your variables. [2pts]
b) Graph the equation on [0, 10, 2] x [0; 400,000; 100,000]. [4pts]
c) Use the equation to determine $N(11)$. Round to the nearest whole number. Write the meaning. [3pts]
d) Use the equation to determine the doubling time of the virus. Solve algebraically. Round to two decimal places. Write the meaning. [6pts]

Student Work

i) $y = ab^x$

$y = 1614\,(1.8465)^x$

$\therefore N(t) = 1614(1.8465)^t$ ✓

ⓑ Graph inserted after.*

ⓒ $N(11) = 1614(1.8465)^{11}$

$= 1373\,238$. (to nearest whole number)

Meaning: After 11 months (in November) ✓ the number of COVID-19 cases world wide is 1 373 238 cases. ✓

ⓓ doubling time:

initial; $N(0) = 1614(1.8465)^0$

$= 1614$ ✓

double: $1614 \times 2 = 3228$

doubling time = $3228 = 1614\,(1.8465)^t$ ✓

$3228/1614 = 1.8465^t$

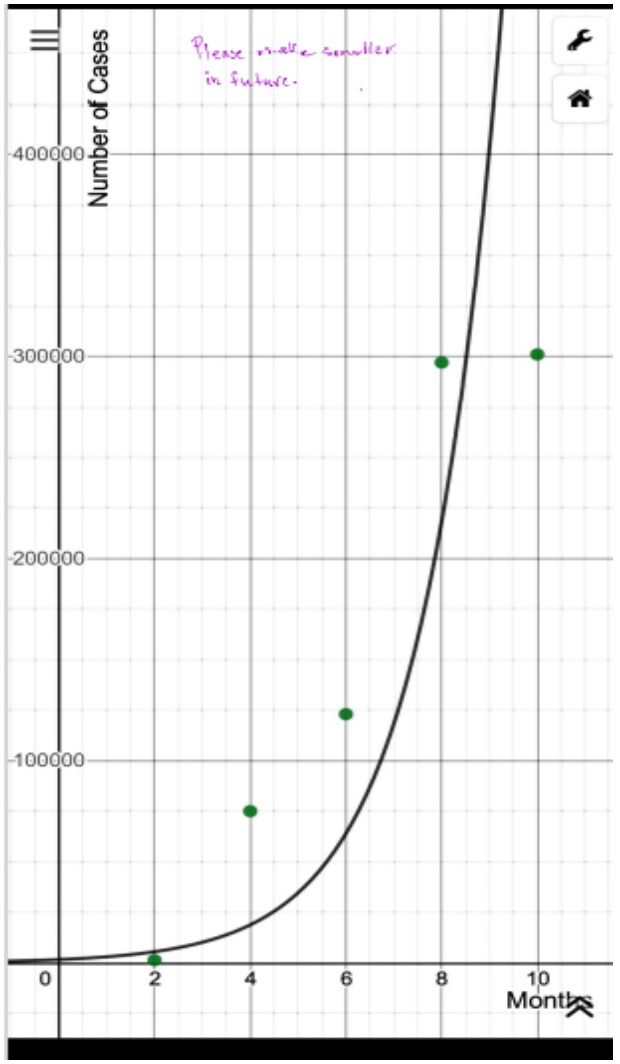
$\ln 2 = \ln 1.8465^t$ ✓

$\ln 2 = t \ln 1.8465$

$\ln 2/\ln 1.8465 = t$ ✓

$\therefore t = 1.13$ ✓ ✓

Meaning: The doubling time of the virus is about 1.13 months ~~days~~



Sinusoidal Model

The Hudson's Bay Company started trading in animal furs in Canada in 1670. They have records dating back to the 1840's. The table shows the population of the wild cat lynx for various years.

Year	**1850**	**1852**	**1854**	**1856**	**1858**
Lynx Population	1100	2300	2900	1900	800

a) Use the calculator to determine the sinusoidal model equation that represents the data. Let t = 0 be the year 1850. Round to two decimal places. Use P and t for variables. [4pts]

b) Sketch the function on [0, 10, 2] x [0, 4000, 1000]. Indicate scale and label axes with correct units. [4pts]

c) Determine the amplitude, period, phase shift and vertical shift of the function. Round to two decimal places. [4pts]

d) Use the equation to determine $P(3)$. Round to a whole number. Write the meaning (give the year). [4pts]

e) Use the equation to determine a year that the population was 2000. Solve algebraically. Write the meaning. [8pts]

Student Work

1 (a) $y_1 = a\sin(bx+c)+d$

$P(t) = \cancel{1076}\ 1073.63\sin(0.64x - 0.79) + 1824.90$ ✓

(c) (i) amp = 1073.63 ✓

(ii) period $= \frac{2\pi}{\omega}$

$= \frac{2\pi}{0.64}$

$= 9.82$ ✓

(iii) P.S $= 0.64x - 0.79 = 0$

$0.64x = 0.79$

$x = {}^{0.79}/_{0.64}$

$x = 1.23$ ✓

$\therefore$ P.S $= 1.23$

(iv) VS $= 1824.90$ ✓

(b) graph inserted in 1 page ✓

(d) $P(3) = 1073.63\sin(0.64(3) - 0.79) + 1824.90$

$= 2795.90 \approx 2796$ ✓

→ This means that in the year 1853, the lynx population ~~will~~ was 2796 ✓

(e) $2000 = 1073.63\sin(0.64x - 0.79) + 1824.90$ ✓

$2000 - 1824.90 = 1073.63\sin(0.64x - 0.79)$

$175.1 = 1073.63\sin(0.64x - 0.79)$ ✓

${}^{175.1}/_{1073.63} = \sin(0.64x - 0.79)$ ✓

$\sin^{-1}\left({}^{175.1}/_{1073.63}\right) = 0.64x - 0.79$ ✓

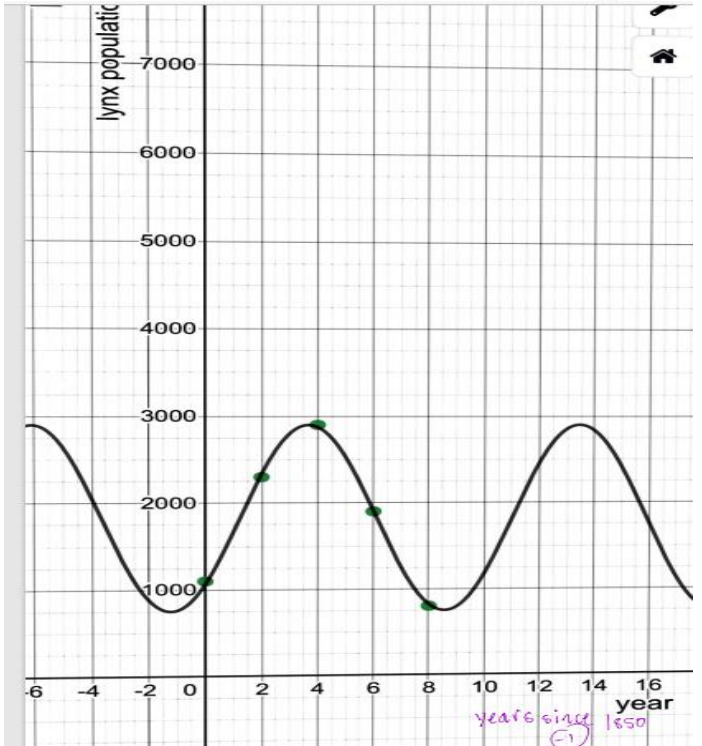
$\sin^{-1}\left({}^{175.1}/_{1073.63}\right) + 0.79 = 0.64x$ ✓

$\frac{\sin^{-1}\left({}^{175.1}/_{1073.63}\right) + 0.79}{0.64} = x$ ✓

$\therefore x = 1.49$

Good!

→ This means that the population was 2000 between the years ~~1850 and~~ 1851 and 1852 ✓



Fall 2021 Precalculus Courses

My Section of Precalculus

- 19 out of 22 students (86.4%) passed with ABC

All Sections of Precalculus at MC

- 420 students – 65.5% passed with ABC

Fall 2020 Precalculus

My Two Sections of Precalculus

- 40 out of 54 students (74.0%) made ABC
- 26 of the 30 students (86.7%) who went on to take Calculus I, made ABC

All Sections of Precalculus at MC

- 67.6% of 565 students made ABC
- 68.9% of all 264 students who went on to take Calculus I, made ABC

My Fall 2020 Precalculus Courses

- 40 of the 54 students (74.0%) made ABC
- 26 of the 30 students (86.7%) who went on to take Calculus I, made ABC
- 11 students went on to take Calculus II made ABC
- 1 student made ABC in Calculus III and Linear Algebra

My Experience

- It was nice to slow down and not rush so many topics
- Students weren’t rushed and had time to present quality work
- Students developed wonderful problem-solving skills
- Students saw where precalculus applies in the real-world
- Precalculus was a good experience for the students

Student Comments about My Strategy

Romolio: With this approach I felt like the flow was great and it helped me with my confidence and understanding. I feel better about math today than I did in the beginning. Starting with modeling made it easy to grasp the other concepts and made the flow manageable

Andre: Compared to previous classes, I liked it. It helped me to understand the material better and actually know what I'm doing in the course rather than just a rough understanding of what to do to pass

Caroline: I heard how hard pre calc was before taking the class, and I was nervous that I would get overwhelmed. However I personally felt like the material was manageable and it broke down the real world uses which I enjoyed a lot!

Student Comments about My Strategy

Angelina: Compared to my previous math classes, I felt there was a good flow. I wasn't overwhelmed like I usually am with math. I was able to understand whatever you asked that day and then apply it to homework and all the practices. So I think the method you chose to teach was very affective.

Jaimie: I definitely appreciate the application side of math like in all the models. Personally, I just find word problems difficult. Whether its interpreting what is being asked, and setting up my formulas based on that, which your approach definitely helped, giving us the variables and filling in the formula the way we needed to. It got easier as the semester went on. The first test, the first word problem, the first application, I thought "Jeez, this is such a pain." It definitely got easier, and I appreciate that.

Teaching Change, by Jose Antonio Bowen, p.296

"Teachers never have enough time…The choice, however is clear. We can cover more, knowing that its impact will not last much past the final exam, or we can redesign to teach less content and more thinking."

Thank you to my wonderful, caring SET colleagues!

- Joan
- Marie
- Katya
- Sowmi
- Carrie
- Heather
- Tito
- Omowale
- Alicia
- Celia
- Pat
- Michele
- Ginger
- Sirisha